Chapter 03: Computer Arithmetic

Lesson 01:

Representations of Positive and Negative Integers

Objective

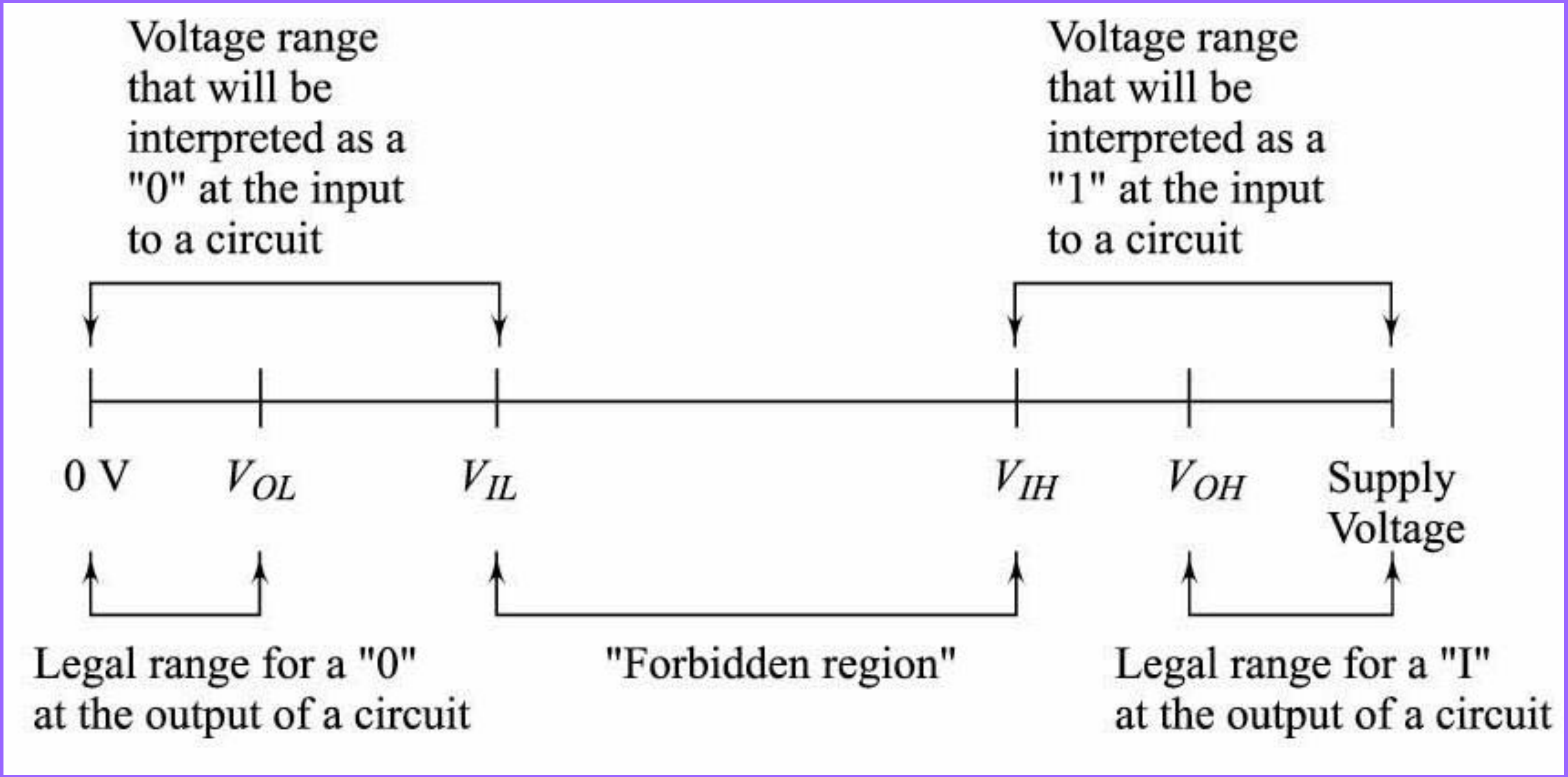
- Understand the representations of positive and negative integers
- Understand the representations for unsigned and signed integers
- Decimal, Binary and Hexadecimal Numbers
- Positive Only Integers Numbers
- Sign-magnitude representations
- Two's complement Representation
- Finding 2's complement

Digital system's signaling convention

Digital system's signaling convention

- Determines how analog electrical signals are interpreted as digital values 0 or 1
- Mapping of to Bits can be in terms of either a Voltage, current, frequency, phase in analog electrical signals

Mapping of Voltages to Bits



Binary, hexadecimal and decimal representation

Convention Postfix b and h

- "b" postfix to identify them as binary, rather than decimal numbers
- “h" postfix to identify them as hexadecimal, rather than decimal numbers

Binary Numbers

- In base-10 arithmetic, numbers represented as the sum of multiples of each power of 10, so the number 1543 = (1 ×103) + (5 × 102) + (4 × 101) + (3 × 100)
- Positive integers are represented using a place-value binary (base-2) system— similar to the place-value system used in decimal (base-10) arithmetic

Binary, hexadecimal and decimal representation examples

- $00100111_b = (0 \times 2^7) + (0 \times 2^6) + (1 \times 2^5) + (0 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) = 39$
- $27_h = (2 \times 16^1) + (7 \times 16^0) = 39_d$

Convention Prefix 0b and 0x

- "0b" to identify them as binary, rather than decimal numbers
- "0x" to identify them as hexadecimal, rather than decimal numbers

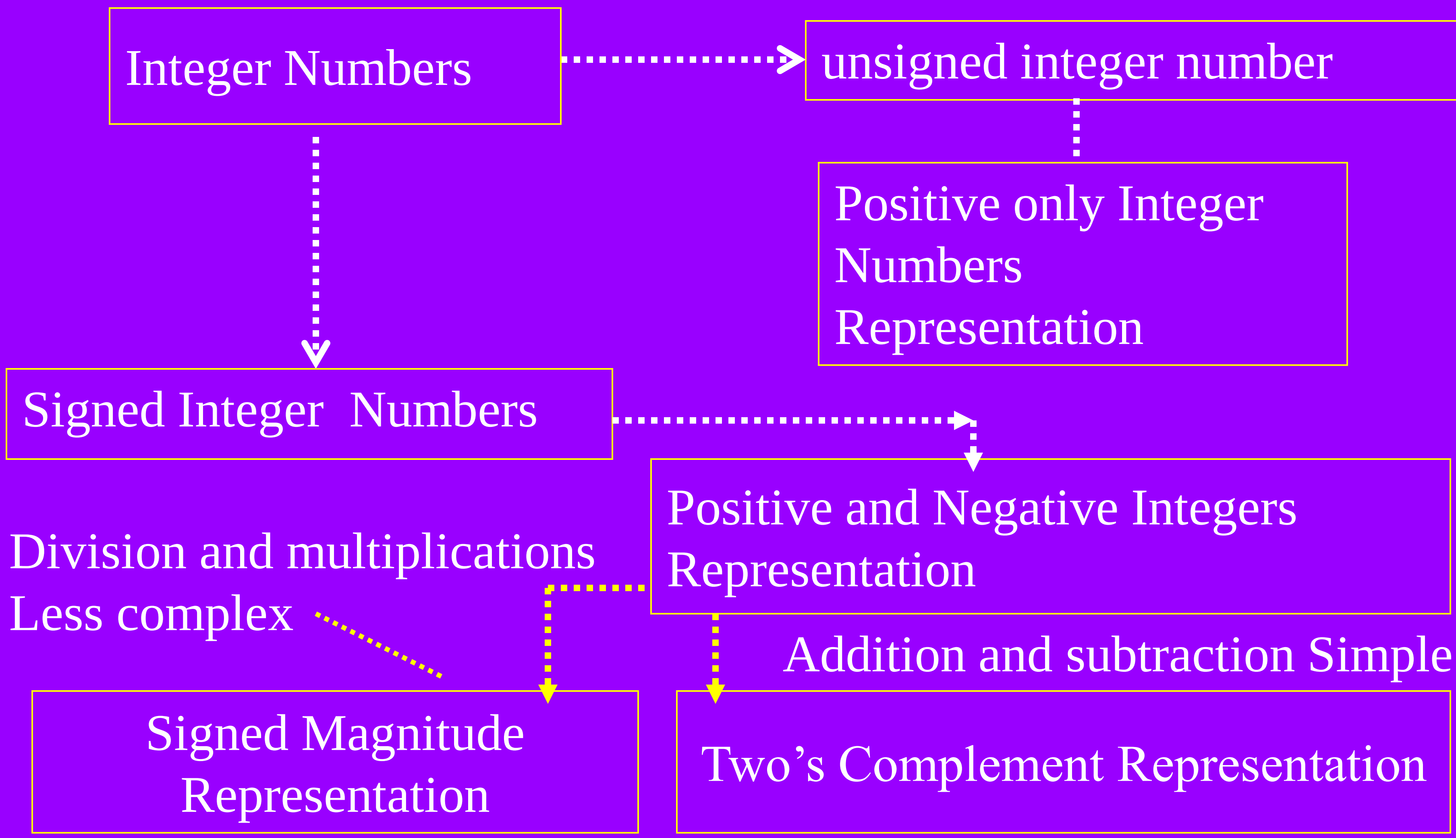
Decimal, Binary and Hex numbers

- 0 0b0000 or 0000_b 0x0 or 0_h
- 4 0b0100 or 0100_b 0x4 or 4_h
- 9 0b1001 or 1001_b 0x9 or 9_h
- 13 0b1101 or 1101_b 0xD or D_h
- 15 0b1111 or 1111_b 0xF or F_h

Hexadecimal Notations

Decimal Number	Binary Representations	Hexadecimal Representation
0	0b0000	0x0
1	0b0001	0x1
2	0b0010	0x2
3	0b0011	0x3
4	0b0100	0x4
5	0b0101	0x5
6	0b0110	0x6
7	0b0111	0x7
8	0b1000	0x8
9	0b1001	0x9
10	0b1010	0xA
11	0b1011	0xB
12	0b1100	0xC
13	0b1101	0xD
14	0b1110	0xE
15	0b1111	0xF

Integer Number Representations



Positive Only Integers (unsigned integers)

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- Positive integers represented using a place-value binary (base-2) system with msb also having a place value
- 0b00100111 = $(0 \times 2^7) + (0 \times 2^6) + (1 \times 2^5) + (0 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) = 39$
- msb 0 is also having a places value
- 0b00100111 = $(1 \times 2^7) + (0 \times 2^6) + (1 \times 2^5) + (0 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) = 128_d + 39_d = 167_d$

Positive Only Integers (8-bit unsigned integers)

- 0b11100111 = $(1 \times 2^7) + (1 \times 2^6) + (1 \times 2^5) + (0 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) = 231$
- 8 bit unsigned number can be between 0 and 255— 0 and 2^{8-1}

Positive Only Integers (16-bit unsigned integers)

- 0b1000000011100111 = $(1 \times 2^{15)} +$ $(1 \times 2^7) + (1 \times 2^6) + (1 \times 2^5) + (0 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) = 32999$
- 16 bit unsigned number can be between 0 and 65535— 0 and 2^{16-1}

Positive Only Integers (32 bit- unsigned integers)

- $0x10000000 = (1 \times 16^7) + (0 \times 16^6) + (0 \times 16^5) + (0 \times 16^4) + (0 \times 16^3) + (0 \times 16^2) + (0 \times 16^1) + (0 \times 16^0) = 268435456$
- 32 bit unsigned number can be between 0 and 4294967295, (0 and 2^{32-1} or 0 and $16^7 - 1$)

Representing of Positive and negative Numbers (Signed Numbers)

Representing of Positive and negative Numbers (Signed Numbers)

- Sign-magnitude representation uses msb (maximum significant bit) = 0 for the +ve number and 1 for -ve number)

0b00100111

msb

sign

lsb

Sign magnitude integer number

- Positive integers represented using a place-value binary (base-2) system with msb do not have a place value
- 0b00100111 = + [$(0 \times 2^6) + (1 \times 2^5) + (0 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$] = 39

Sign magnitude integer number

- Positive integers represented using a place-value binary (base-2) system with msb do not have a place value
- msb = 1— –ve number
- = 0— + ve number
- 0b10100111 = $-(0 \times 2^6) + (1 \times 2^5) + (0 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) = -39$

Sign magnitude integer number

- 0b11100111 = $-[(1 \times 2^6) + (1 \times 2^5) + (0 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)] = -103$
- 8 bit signed number in signed value representation can be between + 0 and + 127 and – 0 and – 127
- + 0 and – 0 is same— 0b10000000 and 0b00000000 same

Sign magnitude integer number (16-bit signed integer)

- 0b1000000011100111 = –231 =
- 16 bit signed number can be between + 0 and + 32767 and – 0 and – 32767

Sign magnitude representation Integers (32 bit- signed integers)

- 0x8000000A = $-(0 \times 16^6) + (0 \times 16^5) + (0 \times 16^4) + (0 \times 16^3) + (0 \times 16^2) + (0 \times 16^1) + (10 \times 16^0) = -10$
- 32 bit signed number can be between +0 and +268435455 and

–0 and – 268435455

Two's complement Representation

Two’s Complement Negation

Original value:	0b00001100 (12)	
Negate each bit:	0b11110011	(Two's-complement
Add 1:	0b11110100	representation of-12)

Two's complement representation for integer number

- Positive integers represented using a place-value binary (base-2) system with msb do not have a place value
- 0b00100111 = $+ [(0 \times 2^6) + (1 \times 2^5) + (0 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)] = +39$

Two's complement Number as Signed Number

- Two's complement representation gives msb (maximum significant bit) = 0 for +ve number and 1 for -ve number)
- **But** if msb = 1, then number is negative and value is as per its two's complement

Two;s complement integer number

- Negative integers represented using a place-value binary (base-2) system with msb don't have a place value and – ve number is found when msb = 1 from two's complement
- 0b11111100 = – [Two's complement of 1111100] = – 4

Examples in Two's complement representation

- $0000\ 0000_b$ 0_d
- $0000\ 0001_b$ $+1_d$
- $0000\ 1100_b$ $+12_d$
- $0100\ 0001_b$ $+65_d$
- $0111\ 1110_b$ $+126_d$
- $0111\ 1111_b$ $+127_d$

Examples in Two's complement representation

- $1111\ 1111_b$ -1_d
- $1111\ 1110_b$ -2_d
- $1111\ 1100_b$ -4_d
- $1000\ 0000_b$ -128_d

Two's complement integer number

- 8 bit two's complement integer number can be between 0 and + 127 and – 1 and – 128
- 0b10000000 and 0b00000000 not same in two's complement representation

Two's complement integer number (16-bit signed integer)

- 0b1111111111111000 = –8
- 16 bit signed number can be between + 32767 and – 32768

Steps in finding Two's complement representation

- If msb = 0, then remaining bits give a +ve value
- If msb = 1, then all bits represent a -ve number with values found from 2's complement as follows:

2's complement

- 0b 1111 1111 ⟶ 0000 0000 (Finds 1's complement by inversion)
- 0000 000 1 (Increment by 1)
- Number is Negative and is –1

2's complement

- 0b 1000 0011 ⟶ 0111 1100 (Finds 1's complement by inversion)
- 0111 1101 (Increment by 1)
- Negative Number –125

2's complement

- 0b 1000 0000 ⟶

  0111 1111 (Finds 1's complement by inversion)

  1000 0000 (Increment by 1)

Negative Number is –128.

2's complement

- 0b 1000 1000

  0111 0111 (Finds 1's complement by inversion)

  0111 1000 (Increment by 1)

Number is –120.

Two's complement of a +ve number gives -ve number

- 0b 0000 0101+ve Number = + 5 as msb = 0
- 1111 1010 (Finds 1's complement by inversion)
- 1111 1011 (Increment by 1)
- Negative Number –5 represented as 1111 1011

Two's complement Two times gives same number back

- Negation twice gives same
- Number 0b 0000 0101 (+5) Two's complement = 1111 1011 (–5)
- Number 0b 1111 1011 Two's complement = 0000 0101 (+5)

Two's complement representation Integers (32 bit- signed integers)

- 0x6000000A = $(6 \times 16^7) + (0 \times 16^6) + (0 \times 16^5) + (0 \times 16^4) + (0 \times 16^3) + (0 \times 16^2) + (0 \times 16^1) + (10 \times 16^0) = (6 \times 16^7) + 10 = +\,268435466$ because msb = 0_b
- 32 bit signed number can be between $+2013265919_d$ and $-\,2013265920_d$

Two's complement representation Integers (32 bit- signed integers)

- 0xFFFFFA = 0b1111 1111 1111 1111 1111 1111 1111 1010. One's complement = 0b0000 0000 0000 0000 0000 0000 0101. Two's complement = 0b0000 0000 0000 0000 0000 0000 0101 + 0b1 = 0x00000006 = – 10
- 32 bit signed number can be between $+2013265919_d$ and -2013265920_d

Summary

We learnt

- Understand the representations for unsigned and signed integers
- Decimal, Binary and Hexadecimal Numbers
- Unsigned n-bit number is between 0 and $+2^n - 1$ for an n-bit representation
- Signed n-bit number in sign-magnitude representation is between 0 and $+ (2^{n-1} - 1)$ and 0 and $- (2^{n-1} - 1)$

We learnt

- Signed n-bit number in two's complement is between + $(2^{n-1} - 1)$ and – (2^{n-1})
- Two's complement is equivalent to negation of a number
- Two's complement is found by first finding 1's complement (inversion) of all bits and then incrementing that by 1

End of Lesson 01 on

Representations of Positive and Negative Integers